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Hiding inside an arbitrarily shaped metal pit using homogeneous metamaterials

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Even though its origins were at science fiction and fantasy, invisibility cloaks are a reality. However, realization of invisibility cloaks requires material characteristics that are hardly available in nature. Fortunately, artificial designer materials known as metamaterials provide unprecedented freedom in synthesizing materials with flexible electromagnetic properties. In this paper, by introducing a new transformation that can warp the space to deflect light inside an arbitrarily shaped metal pit filled with diagonal metamaterial (i.e. a metamaterial with permittivity and permeability tensors that are diagonal), we show that objects can be effectively hidden from view. The implementation requires a single homogeneous metamaterial and thus much different from the existing alternative approaches that require multiple homogeneous metamaterials. To show the utility of the proposed method, using detailed numerical simulations we show two distinct cloaks that can conceal the objects by the same diagonal metamaterial medium and analyze the impact of the losses in the medium on this method.

1. Introduction

Transformation optics (TO) \cite{1–9} relies on warping the space to control the trajectory of light in a medium. If the warping of the propagation space can be done in such a way to deflect light around an object embedded in a material, then light reaching an observer seems to have come straight through by concealing the object as if it were not there \cite{10–12}. Even though its origins were in science fiction and fantasy, such invisibility cloaks are a reality \cite{13,14}. In 2006, Schurig et al. \cite{15} presented an invisibility cloak that successfully demonstrated the use of this concept. In TO, warping space is achieved using a coordinate transformation which nevertheless preserves the structure of Maxwell’s equations. By specifying the permittivity and permeability changed locally in the real space, one can obtain a medium to emulate the warping space suited for TO operations (i.e. to bend light as required). Some other fascinating applications of this principle include field concentrator \cite{16}, beam collimators and shapers \cite{17}, illusion optics \cite{18–20}, and astronomically equivalent black hole \cite{21}. Among all these novel devices, the invisibility cloak which can render whatever it covers unseeable, can be considered the most amazing application and thus attracts much attention \cite{22–24}.

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Although using TO, in principle it is possible to theoretically realize aberrant effects, practical realization typically demands materials with highly anisotropic and inhomogeneous constitutive parameters, not in many instances readily available in nature. However, due to recent advances in materials technology, an extraordinary range of electromagnetic properties can be engineered in artificially structured materials termed metamaterials [25–30], with a flexibility unmatched by any conventional materials. This opens up a plethora of opportunities to synthesize materials suited for TO. The major difficulty for electromagnetic cloaks is the extreme complexity underlying such synthesis and fabrication processes. Therefore, carefully designed spatial transformation leading to simple metamaterial parameters will yield practically more realizable solutions. For example, invisibility cloaks for dc magnetic fields may be achieved by simple arrangements of superconductors and isotropic magnetic materials [31,32]. Great efforts have been exerted to design TO-based devices with homogeneous metamaterials [33]. Luo et al. [34] rigorously analyzed the conical cloak and found that the constitutive parameters could be spatially invariant in each region of the cloak. Li et al. [35,36] presented an invisible cloak and a field concentrator that consisted of eight triangular components with homogeneous materials. Most recently, Zhu et al. [37] proposed that the object in a corner can be hidden by a cloak made of two triangles with homogeneous metamaterials. Although these works provide homogeneous material parameters for TO-based devices, the constitutive parameters are still complicated anisotropic tensors. What is very interesting in all the above cases is that all of them use a set of triangles, each being made using some homogeneous material, limiting the applicability of the technique to other geometries.

In this paper, we present a new coordinate transformation that can warp the space to deflect light around an arbitrarily shaped metal pit filled with diagonal metamaterial. The most appealing aspect of this new transformation is that the constitutive parameters of the metamaterial making up the invisibility cloak are spatially independent with diagonal permittivity and permeability tensors. Strikingly, our transformation is different from the reported works [34–37], where several homogeneous metamaterials are necessary to build the TO devices whereas the cloak we proposed can be synthesized using a real homogeneous metamaterial. Moreover, the transformation is not particular to a specific geometry, and it is possible to conceal objects in a metallic pit of arbitrarily shaped entrance with no special treatment. We illustrate this point by concealing two geometrically distinct cloaks with a designer metamaterial medium via employing numerically full-wave finite-element simulations.

2. Theoretical analysis

Figure 1(a) shows a schematic diagram of the general case under consideration in this paper showing a metal pit with arbitrarily shaped boundary interface, \( f_2(x) - y = 0 \), filled with a

![Figure 1](http://example.com/figure1.png)

Figure 1. (a) The cross section of the arbitrary invisible cloak in the Cartesian coordinate, which transforms the irregular boundary \( f_2(x) \) to \( f_1(x) \); (b) a triangle cloak with \( f_1(x) = kf_2(x) = k|x| - d \); and (c) a semi-elliptical cloak with \( f_1(x) = kf_2(x) = -k \sqrt{d^2 - x^2} \). In each case, the region marked by “metamaterial” is the invisibility cloak constructed with the transformed medium.
metamaterial. For the ease of analysis, without loss of generality, we assume that the pit is identical along the z-axis, and thus restricts variations in the xoy plane in our analysis. If an object is placed inside the pit (which can even be filled with any generic material), the light scattered from the object will be quite different from that directly reflected from the boundary $f_2(x) - y = 0$. In other words, under general conditions, an object placed inside the pit is observable from outside. The object can only be concealed from view if we find a way to suppress the scattering experienced by a probing light beam, which can be achieved by cleverly devising a coordinate transformation to compress the irregular pit region bounded by $f_2(x) - y = 0$ and $y = 0$ surfaces into the orange region bounded by $f_1(x) - y = 0$ and $y = 0$ surfaces, which essentially excludes light interacting with the buried object. The transformation is chosen such that $f_1(x) - y = 0$ surface mimics the $f_2(x) - y = 0$ optical properties with proper amplitude and phase relations so that a probing beam sees no observable differences. That is, under this transformation, an incident light reflected at the boundary $f_1(x) - y = 0$ will behave as if it were reflected at the boundary $f_2(x) - y = 0$ in the original setup. Therefore, the object inside the blue region is well hidden, deemed to be covered by an invisibility cloak. To illustrate this idea more clearly, we also sketch two different cloaks in Figure 1 where Figure 1(b) shows a triangular cloak and Figure 1(c) shows a semi-elliptical cloak.

To quantify the idea, consider the scaling transformation, $f_1(x) = k f_2(x)$, where $0 < k < 1$ is the compression ratio along the y direction. We assume $f_2(x) \leq 0$ at the whole region of $a < x < b$, while $f_2(a) = f_2(b) = 0$ at the two endpoints. In this case, the coordinate mapping can be expressed as:

$$
\begin{align*}
    x' &= x, \\
    y' &= ky, \\
    z' &= z,
\end{align*}
$$

(1)

where the coordinates $(x, y, z)$ represent the original pit without any filling, whose permittivity and permeability tensors are vacuum values $\varepsilon = \mu = I$ ($I$ is the identity matrix), and the coordinates $(x', y', z')$ represent the transformed physical space filled with the corresponding metamaterial. According to the TO theory [1], the constitutive parameters of the transformed metamaterial in the new space can be obtained as a geometric mapping: $\varepsilon' = \mu' = JJ^T / \det J$, where $J$ is the Jacobian with components $J_{ij} = \partial x_i / \partial y_j$ and $\det J$ denotes its determinant. The coordinate mapping in Equation (1) yields the constitutive parameters of the metamaterial for the designed cloak:

$$
\varepsilon' = \mu' = \begin{pmatrix} 1/k & 0 & 0 \\ 0 & k & 0 \\ 0 & 0 & 1/k \end{pmatrix}.
$$

(2)

Compared to the previews works [34–37], three advantages could be found in our design. First, the material parameters for this cloak is spatially independent in the whole cloak region which is very much different from the available published work where multiple such regions are generally required. As a result, we only need a single metamaterial to construct this cloak. The second advantage is associated with having the permeability and permittivity tensors diagonal, which is much desirable compared with symmetrical tensors available in published literature (no further diagonalized procedure is required). The third feature is that the coordinate mapping in Equation (1) is independent of interface shape $f_2(x) - y = 0$, implying that we can design cloaks for arbitrarily shaped pits using the same transformation (see, e.g. Figure 1(b) and 1(c)). It is overwhelmingly clear that these properties will surely reduce the difficulty in fabricating cloaks for this setup suited for various applications.
It is interesting to note that if the incident wave is transverse magnetic (TM) polarized, only $\varepsilon_{xx}$, $\varepsilon_{yy}$, and $\mu_{zz}$ components of the constitutive parameters are necessary to describe the metamaterial suited for filling the metal pit. Therefore, by keeping $\varepsilon_{xx}\mu_{zz}$ and $\varepsilon_{yy}\mu_{zz}$ as constants, the specification in Equation (2) can be further simplified to:

$$
\varepsilon' = \begin{pmatrix} 1/k^2 & 0 \\ 0 & 1 \end{pmatrix}, \quad \mu' = 1.
$$

The above prescription clearly shows for the TM case that the resulting tensors not only are homogeneous and diagonal, but also closely match with free space values except one component, emphasizing the greater practical utility of the proposed method.

### 3. Numerical simulation and discussion

To illustrate the performance of the proposed cloak with the designed constitutive parameters in Equations (2) and (3), we introduce two examples with different geometries. The first example is a triangular cloak as shown in Figure 1(b), where the boundary $f_2(x) = |x| - d$, $(-d \leq x \leq d)$. The cloaking transformation maps (compresses) the space enclosed by this right-angled triangle to a triangle with smaller extent. Our second example is a semi-elliptical cloak where we set $f_2(x) = -\sqrt{d^2 - x^2}$, $(-d \leq x \leq d)$, and the corresponding transformation maps (compresses) the semi-circular space to a semi-elliptical one (Figure 1(c)). In both cases, we set the compression ratio, $k = 1/2$.

Numerical simulations are performed by commercial finite-element package COMSOL Multiphysics (version 4.2) to illustrate the invisibility of these two cloaks. In both case, we set $d=10$ mm and the free-space wavelength of the incident wave is 2 mm. The metal in the

![Figure 2. Snapshots of the magnetic field distributions under TM wave incidences for (a)–(c) the triangular cloak and (e)–(f) semi-elliptical cloak. Here, (a) and (d) show reference configurations without the cloaks and objects. The ideal cloak implementations are given in (b) and (e). Similar results for the simplified cloak (for the TM case) are given in (c) and (f). The semi-circle of radius $r$ in (b) shows schematically where the magnetic field is monitored for Figures 3 and 4.](image)
simulations is copper with the conductivity $\sigma = 6.0 \times 10^7$ S/m. For simplicity, the lower surface of the cloak $f_1(x) - y = 0$ is set as perfect electric conductor boundaries, so as to reflect the total energy of the incident wave at this surface (which can be simply realized in practice by laying a thin metal sheet closely under the cloak, and such a treatment is widely used in carpet cloaks). To show the invisibility of the proposed cloaks under different illuminations, we use a line source which excites a Gaussian plane TM wave for the triangular cloak, whilst in the case of the semi-elliptical cloak, a point source is adopted to drive a TM polarized cylindrical wave.

Figure 2 shows the magnetic field distributions for each cloak interacting with corresponding sources. In Figure 2(a) and (d), to gage the response of the systems to external probing fields, the reference field distributions free of cloaks and objects (to be hidden) are presented. When the ideal cloaks with the designed constitutive parameters (see Equation (2)) are placed in the metal pit, the incident waves in both systems are compressed into the associated metamaterial regions, and the reflected field patterns (Figure 2(b) and (e)) observed outside match the corresponding reference distributions in Figure 2(a) and (d). Therefore, to an outside observer, no change can be seen in the configurations with and without the cloaks, i.e. a right-triangular or semi-circular pit, and the object under the metamaterial will be hidden completely. Similarly, for the simplified cloaks for the TM wave filled with the designer parameters in Equation (3), the magnetic fields in Figures 2(c) and (f) match those of the corresponding reference field distributions in Figures 2(a) and (d).

To better understand the performance of the cloak, we plot the amplitudes of magnetic fields for all the cases in Figure 2 along the semi-circle $r = 6$ mm, $(0 < \theta < 180^\circ)$, where $\theta$ is the angle of the radius with the positive $x$ axis. As can be seen from Figure 3(a) and 3(b), the curves for the direct reflection from the metallic right triangle (semi-circle) and from the ideal cloak are almost overlapped, which implies the perfect performance of invisibility. However, some differences are found for TM incident wave scenario (i.e. the simplified cases corresponding to Equation (3)) where a few low fluctuations arise due to the impedance-mismatch introduced during the simplifications.

Up to this point, all the materials used for the cloaks were assumed to be lossless. However, losses, which are normally unavoidable in metamaterials, may ultimately limit its effectiveness and utility. To investigate the effect of loss on the proposed homogeneous cloaks, we introduce a loss factor $\sigma$ into the constitutive parameters of the designed cloak: $\varepsilon'' = \varepsilon' (1 + i\sigma)$, $\mu'' = \mu' (1 + i\sigma)$, where $\varepsilon''$ and $\mu''$ are the permittivity and permeability of the lossy cloaks. With the inclusion of these parameters, both the permittivity and the permeabil-

![Figure 3](image-url)
ity tensors are modified in the same manner so as to exclude the impact of additional impedance-mismatch. To elaborate on this, we plot the calculated magnetic fields at $r = 6\, \text{mm}$ for different values of $\sigma$ in Figure 4 for the ideal triangular cloak considered in Figure 2(b). From this plot, it is very clear that the magnetic field curves change very little for the region $90^\circ < \theta < 180^\circ$ which mainly corresponds to the incident wave. However, for the reflected wave in the region $0 < \theta < 90^\circ$, the conclusion is quite different. As the loss increases, the magnetic field of the reflected wave drops monotonically and rapidly, implying that invisibility performance degrades as expected. Specifically, the amplitude of magnetic field becomes less than 0.2 even when the value of $\sigma$ is as small as 0.02, in which case the reflected wave is highly suppressed by the losses. To remedy this, one may employ a gain medium to compensate the losses of the metamaterials [38,39], but it will inevitably make fabrication difficult. Nevertheless, if the incident wave is restricted to TM mode, from Equation (3) it is clear that only a non-magnetic biaxial medium is needed to construct the simplified cloak. Most importantly, none of the permittivity element of the biaxial medium is less than one, implying that such a cloak may be realized by using a non-resonant medium or a natural occurring crystal [40–42]. It should be noted that the permittivity of the required material needs an anisotropy factor of four for the cloaks in Figure 2, which is far greater than what natural crystals can provide. However, one can reduce such an anisotropy factor to any desired value by choosing a proper compression ratio $k$ (i.e. closer to 1).

4. Conclusion

In conclusion, the TO offers great tools of designing novel optical devices that can manipulate light beyond what is offered by nature. However, many such designs are difficult to realize in practice due to fabrication difficulties. In this paper, we present novel spatial transformations for realizing the invisibility cloaks for objects hidden inside a metal pit. The most striking aspect of our simple transformation is that it enables one to realize the invisibility cloak using a homogeneous metamaterial medium with diagonal susceptibility tensors. To illustrate the versatility of our method, we design two invisible cloaks with different geometries but can be constructed with the same metamaterial and the invisibility performances which were demonstrated via full-wave simulations. Similar transformations may also be employed to design other applications such as beam concentrators and expanders, which are beyond the scope of this paper.

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