Dispersion relation for surface plasmon polaritons in metal/nonlinear-dielectric/metal slot waveguides

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We present the first (to our knowledge) exact dispersion relation for the transverse-magnetic surface plasmon polariton (SPP) modes of a plasmonic slot waveguide, which is formed by a nonlinear Kerr medium sandwiched between two metallic slabs. The obtained relation is then simplified to the case of small field intensities, while retaining nonlinear terms, to derive approximate dispersion equations for the symmetric and antisymmetric SPP modes. © 2011 Optical Society of America

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In many nanophotonics applications, there is a need to transmit and manipulate optical energy with deep subwavelength lateral confinement [1]. These tasks may be solved by using different types of plasmonic waveguides—one of the simplest and most efficient of which involves a thin dielectric layer embedded between two sufficiently thick (with thicknesses in the order of tens of nanometers) layers of metal [2–4]. If the dielectric material exhibits a nonlinear response at optical frequencies, the power flow along the plasmonic waveguide can be conveniently controlled with such nonlinear effects as self-focusing [5–7], self-phase modulation [8], four-wave mixing [9], and second-harmonic generation [10]. Any attempt to employ the advanced features offered by nonlinear plasmonic waveguides at high optical powers should begin with determining what effect nonlinearity in the dielectric layer would have on the dispersion of surface plasmon polaritons (SPPs). Despite the clear advantage of using SPP dispersion relations in an analytic form without having to find them through the numerical solution of Maxwell’s equations when analyzing the performance of plasmonic waveguides, such relations are not available in the literature for metal/nonlinear-dielectric/metal slot waveguides. In this Letter, we will fill said gap by deriving the required dispersion equation in quadratures and applying it to the special case of small nonlinear changes in permittivity, in order to obtain closed-form dispersion relations for the SPP modes of different symmetries.

Consider a transverse-magnetic (TM) mode of a lossless metal/dielectric/plasmonic waveguide, whose dielectric layer is a Kerr medium characterized by the linear permittivity $\varepsilon_{1L}$ and nonlinear coefficient $\alpha$. Let the electric field of the mode,

$$E(x, z) = [E_x(x)e_x + iE_z(x)e_z] \exp(i\beta z),$$

be described in terms of the real functions $E_x(x)$ and $E_z(x)$, which satisfy the equations

$$\frac{dE_x}{dx} = \frac{k^2}{\beta} E_x, \quad \frac{dE_z}{dx} = \beta \varepsilon_2 E_z,$$  \hspace{1cm} (1)

where $k_j = (\beta^2 - \varepsilon_j k^2)^{1/2} > 0$ is the attenuation factor in the $j$th medium, $k$ is the free-space wavevector and we assume real propagation constant $\beta$ and real permittivities $\varepsilon_j$ for dielectric ($j = 1$) and metal ($j = 2$). In the region $x > d/2$ occupied by the metal (see Fig. 1), the solution of these equations may be written as

$$E_{x2} = A \exp(-k_2 x), \quad E_{z2} = -(k_2/\beta) A \exp(-k_2 x),$$

where, without loss of generality, we take constant $A$ to be negative. The continuity of the electric displacement vector and component $E_z$ across the interface $x = d/2$ yields

$$\varepsilon_1^+ E_{x1}^+ = \varepsilon_2 A \exp(-k_2 d/2),$$

$$E_{z1}^+ = -(k_2/\beta) A \exp(-k_2 d/2),$$

where $\varepsilon_1^+ = \varepsilon_{1L} + \alpha(E_{x1}^+)^2$, $(E_{x1}^+)^2 = (E_{z1}^+)^2 + (E_{z1}^+)^2$, and the parameters $E_{z1}^+$ are implicitly defined. These equations allow us to relate the electric field components at the interface being considered as

$$E_{x1}^+ = E_{z1}^+ \sqrt{1 + (E_{z1}^+)^2 (\beta/k_2)^2}. \hspace{1cm} (2)$$

In order to obtain the dispersion equation, we need to know the function $E_{z1}(E_{z1})$ everywhere inside the nonlinear medium. It can be found by noticing that Eq. (1) leads to the identity

Fig. 1. (Color online) Representative electric field profiles for the (a) symmetric and (b) antisymmetric SPP modes of a plasmonic slot waveguide, which is created by a nonlinear-dielectric layer of permittivity $\varepsilon_1$ and thickness $d$ placed between two metallic slabs of permittivity $\varepsilon_2$. © 2011 Optical Society of America
with \( \varepsilon_1 = \varepsilon_{1L} + \alpha \varepsilon_0^2 \) and \( \varepsilon_1' = \varepsilon_{1L}^2 + \varepsilon_0^2 \). Integrating this identity with respect to \( x \), we arrive at the conservation law \[ C(\varepsilon_{x1}) = \left( \frac{\varepsilon_1(\varepsilon_{x1}) - \varepsilon_{1L} - \varepsilon_{x1}^2}{3} \right)^{1/2}, \quad (4) \]

with \( \varepsilon_1(\varepsilon_{x1}) = \frac{2\alpha \varepsilon_0^2 + \sqrt{(2\alpha \varepsilon_0^2)^2 + C(1 + 2\varepsilon_0^2)}}{1 + 2\varepsilon_0^2} \).

Notice, that the presence of the factor \( \alpha \varepsilon_0^2 \) in Eq. (3) prohibits restoration of the nonlinear conservation law from its linear analog through the replacement \( \varepsilon_{1L} \to \varepsilon_1 \).

This is why a similar replacement performed in the linear dispersion equation would fail to provide the correct dispersion relation in the nonlinear case.

The value of constant \( C \) may be expressed through the electric field intensity, \( E_0 \), at the mode center. For the symmetric mode shown in Fig. 1(a), with \( E_{x1}(x) = E_{x1}(-x) \) and \( E_{x1}(0) = 0 \), Eq. (3) yields \[ C(E_0) = \varepsilon_{10}|\varepsilon_{10} - 2(2 - \chi \varepsilon_0^2)\alpha \varepsilon_0^2|, \quad (5) \]

where \( \varepsilon_{10} = \varepsilon_{1L} + \alpha \varepsilon_0^2 \). In the case of the antisymmetric mode shown in Fig. 1(b), for which \( E_{x1}(x) = -E_{x1}(-x) \), the constant is \[ C(E_0) = \varepsilon_{10}^2. \quad (6) \]

Our derivation is finalized by the integration of Eq. (1) using Eq. (4), which leads to the dispersion relation \( \beta(k, E_0) \) in the implicit form \[ \frac{\beta d}{2} = \int_{E_{x1}(0)}^{E_{x1}(K_0)} \frac{\varepsilon_1(E_x) + 2\alpha \varepsilon_0^2}{E_x(E_x)} \sqrt{(2\alpha \varepsilon_0^2)^2 + C(1 + 2\varepsilon_0^2)} dx. \quad (7) \]

The lower limit in this integral is equal to \( E_0 \) for the symmetric mode and zero for the antisymmetric mode. The upper limit, \( E_{x1}^+(E_0) = \sqrt{\frac{\varepsilon_1^+ - \varepsilon_{1L}}{\alpha(1 + \psi(\varepsilon_1^{+2})^2)}}, \quad (8) \]

may be found by calculating \( \varepsilon_1^+ \) from the fourth-order algebraic equation \[ \varepsilon_1^+ \left( \frac{2(\chi \varepsilon_1^+ - 2)(\varepsilon_1^+ - \varepsilon_{1L})}{\psi(\varepsilon_1^+)^2 + 1} + \varepsilon_1^+ \right) = C(E_0). \quad (9) \]

with \( \psi = k_0^2/(\varepsilon_2^2) \), which is readily derived from Eqs. (2) and (3).

Equations (4)–(9) solve the problem of relating the frequency and wavector of SPPs for a given electric field intensity at the center of the nonlinear waveguide; they constitute the main result of this paper.

It is quite instructive to recover from Eq. (7) the standard dispersion equations for SPPs in linear plasmonic waveguides when \( \alpha \to 0 \). In this limit, Eq. (4) reduces to \( E_{x1}(E_1) = (k_1/\beta)\sqrt{E_{x1}^2 - E_0^2} \) for the symmetric mode and \( E_{x1}(E_1) = \sqrt{E_0^2 + (k_1/\beta)^2 E_{x1}^2} \) for the antisymmetric mode; from this point onwards, it is assumed that \( k_1 = (\beta^2 - \varepsilon_{1L}k_0^2)^{1/2} \). Substitution of these expressions into Eq. (7) and evaluation of the integrals leads to the following relations for the symmetric and antisymmetric modes, respectively:

\[ \cosh(k_1 d/2) = E_{x1}^+/E_0, \quad (10) \]
\[ \sinh(k_1 d/2) = (k_1/\beta)(E_{x1}^+/E_0). \quad (11) \]

Now refer back to Eqs. (5), (6), and (9) and note that, to the first order in \( \alpha \varepsilon_0^2/\varepsilon_{1L} \), we may write \[ \varepsilon_1^+ = \varepsilon_{1L} + \frac{1 + \psi \varepsilon_{1L}^2}{1 - \chi \varepsilon_{1L} - \psi \varepsilon_{1L}^2} \alpha \varepsilon_0^2 \]

for the symmetric mode and \[ \varepsilon_1^+ = \varepsilon_{1L} + \frac{1 + \psi \varepsilon_{1L}^2}{1 - \varepsilon_{1L} - \psi \varepsilon_{1L}^2} \alpha \varepsilon_0^2 \]

for the antisymmetric mode, so that Eq. (8) gives \[ \left( \frac{E_0}{E_{x1}^+} \right)^2 = 1 - \frac{\varepsilon_{1L}^2}{1 - \chi \varepsilon_{1L}} = 1 - \left( \varepsilon_{1L} \frac{k_2}{\varepsilon_2} \right)^2, \]

in the first case and \[ \left( \frac{E_0}{E_{x1}} \right)^2 = \psi \varepsilon_{1L}^2 - 1 + \chi \varepsilon_{1L} = \left( \frac{\varepsilon_{1L} k_2}{\varepsilon_2} \right)^2 - \frac{k_0^2}{\beta^2}, \]

in the second case. Using these results in Eqs. (10) and (11), yields the common dispersion relations \[ \tanh\left( \frac{k_1 d}{2} \right) = \frac{k_1 \varepsilon_{1L} k_2}{|\varepsilon_2 k_1|}, \quad (12) \]

in which, as before, \( \pm \) signs correspond to the symmetric and antisymmetric modes, respectively.

In the special case of weak optical nonlinearity or low optical powers, such that \( \alpha \varepsilon_0^2 \ll \varepsilon_{1L} \), closed-form expressions may be readily obtained from Eq. (7). This can be done by expanding the integrands and upper integration limit in a Fourier series, and keeping the lowest-order terms proportional to \( \alpha \). Some algebra yields a dispersion equation for the symmetric mode of the form
The derived relations are illustrated in Fig. 2 by the example of the symmetric SPP mode, whose dispersion branch does not exhibit a cutoff. The material parameters are chosen to be \( \varepsilon_{1L} = 2.25, \alpha = 2 \times 10^{-16} (\text{m/V})^2, \) \( \varepsilon_2(\omega) = 1 - (\omega_p/\omega)^2, \) \( \omega_p = 1.36 \times 10^{16} \text{Hz}, \) and \( d = 100 \text{nm}. \) The figure shows dispersion curves for \( E_0 = 3 \times 10^7 \text{V/m}, \) which corresponds to the nonlinear change in permittivity of about 8\%. It is seen that the approximate dispersion relation obtained using Eq. (13) (red curve) agrees well with the exact relationship given in Eq. (7) (blue curve) for small propagation constants, \( \beta d < 2.5. \) For larger \( \beta, \) the true dispersion branch bends downwards, while the approximate one continues to follow the dispersion relation in the linear case, plotted by the green curve. A negative slope of SPP dispersion for \( \beta d > 4.5 \) implies that the group and phase velocities are antiparallel, which is due to the fact that the total energy flow of the SPP mode is negative. Insets in Fig. 2 show how the profiles of \( E_x \) and \( E_z, \) obtained with Eqs. (4) and (7), compare with each other for the two values of \( \beta d \) corresponding to the frequency of \( 0.2 \omega_p. \) Since the mode traveling backward is associated with much stronger electric fields than the mode traveling forward, it is in practice difficult, even impossible, to excite SPPs with large wavevectors.

In conclusion, we have derived dispersion relations for the SPP modes of a metal/nonlinear-dielectric/metal slot waveguide, which can be used for modeling integrated plasmonic components with Kerr nonlinearity.

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References