Coherence-vortex lattice formed via Mie scattering of partially coherent light by several dielectric nanospheres

Madara L. Marasinghe,1,* David M. Paganin,2 and Malin Premaratne3
1Advanced Computing and Simulation Laboratory (ACsL), Department of Electrical and Computer Systems Engineering, Faculty of Engineering, Monash University, Clayton, Victoria 3800, Australia
2School of Physics, Faculty of Science, Monash University, Clayton, Victoria 3800, Australia
*Corresponding author: madara.marasinghe@monash.edu

Received December 23, 2010; accepted February 4, 2011; posted February 11, 2011 (Doc. ID 140884); published March 10, 2011

We previously demonstrated that Mie scattering of stationary partially coherent light by dielectric spheres generates coherence vortices. In this Letter, we demonstrate that a lattice of coherence vortices can be generated by Mie scattering of partially coherent electromagnetic waves by a system of three coplanar dielectric spheres. Spontaneous coherence-vortex creation and destruction is observed in our computer modeling of this system. © 2011 Optical Society of America

OCIS codes: 030.1640, 050.4865, 260.6042, 290.4020, 290.5850.

From coherent optical speckle fields [1] to angular-momentum eigenstates of hydrogenic atoms [2], from turbulent Bose–Einstein condensates [3] to the focal volume of aberrated lenses [4], spontaneously nucleated phase vortices exist in abundance in the natural world. These examples illustrate the point that phase vortices, namely screw-type topological defects in the phase φ ≡ arg ψ of a complex wave-function or order-parameter field, ψ [5], constitute a form of natural pattern formation that does not require deliberate “stirring” or specific optical engineering to be formed [6]. A common feature of all vortex structures mentioned above is that they are threaded by nodal lines (branch lines, lines of complete destructive interference) in three-dimensional (3D) space, which terminate either on potential discontinuities or at infinity [7]. The topological charge, m, of such a nodal line can be obtained by integrating around a small simple closed loop, Γ, threading the line, via [5]

\[ m = \frac{1}{2\pi} \oint_{\Gamma} \nabla_{\mathbf{r}_1} \arg[\psi(\mathbf{r}_1)] \cdot d\mathbf{t}_1, \]  

where |ψ| > 0 at each point on Γ, \( \nabla_{\mathbf{r}_1} \) is the gradient operator with respect to \( \mathbf{r}_1 \), and \( \mathbf{t}_1 \) is the unit tangent vector at each point \( \mathbf{r}_1 \in \Gamma \).

Screw-type topological defects [8,9] also exist in the second-order theory of partially coherent fields [10]. Both domain-wall [11] and screw-type [12] defects have been studied in the phase of the mutual coherence function or spectral degree of coherence. Such defects have been studied in both the scalar [8] and vector [13] theory of partial coherence.

Restricting consideration to coherence vortices of statistically stationary electromagnetic fields in the context of the space–frequency description of partial coherence [14], one has the spectral degree of coherence [15]

\[ \mu(\mathbf{r}_1, \mathbf{r}_2; \omega) = \frac{\text{Tr}[W_{ij}(\mathbf{r}_1, \mathbf{r}_2; \omega)]}{\sqrt{S(\mathbf{r}_1, \omega)S(\mathbf{r}_2, \omega)}}, \]  

where \( W_{ij}(\mathbf{r}_1, \mathbf{r}_2; \omega) = \langle E_i^*(\mathbf{r}_1, \omega)E_j(\mathbf{r}_2, \omega) \rangle \) is the cross-spectral density matrix, \( i, j \) denote Cartesian components, \( x, y, z, r_{1,2} \) are position vectors in 3D space, Tr denotes trace, \( \omega \) denotes angular frequency, angular brackets denote ensemble averaging, and \( S(\mathbf{r}_{1,2}; \omega) = W(\mathbf{r}_{1,2}, \mathbf{r}_{1,2}; \omega) \) denotes spectral density.

Given that \( \mu(\mathbf{r}_1, \mathbf{r}_2; \omega) \) is continuous and single-valued, for all pairs of points \( \{ \mathbf{r}_1, \mathbf{r}_2, \omega \} \) such that neither \( \mathbf{r}_1 \) nor \( \mathbf{r}_2 \) coincides with a point of discontinuity in the scattering medium, Eq. (1) generalizes to [13]:

\[ m = \frac{1}{2\pi} \oint_{\Gamma} \nabla_{\mathbf{r}_1} \arg[\mu(\mathbf{r}_1, \mathbf{r}_2; \omega)] \cdot d\mathbf{t}_1, \]  

where \( \mathbf{r}_2 = \mathbf{r}_2' \) and \( \omega = \omega' \) are considered fixed, with a similar formula applying to the case where \( \mathbf{r}_1 \) and \( \omega \) are considered fixed. The physical meaning of the nodal lines, which constitute the branch points of \( \arg[\mu(\mathbf{r}_1, \mathbf{r}_2; \omega')] \) that thread the cores of coherence vortices [13], is that they correspond to pairs of spatial points \( \{ \mathbf{r}_1, \mathbf{r}_2' \} \) for which the partially coherent disturbance is uncorrelated at spatial frequency \( \omega' \) [13].

For fixed \( \mathbf{r}_2 = \mathbf{r}_2' \) and \( \omega = \omega' \), the coherence-vortex defects have their cores threaded by one-dimensional (1D) nodal lines. If \( \omega \) is allowed to vary through a continuum of values, these nodal lines will trace out two-dimensional sheets in the four-dimensional space coordinatized by \( \mathbf{r} = \mathbf{r}_1 \) and \( \omega \), with \( \mathbf{r}_2 = \mathbf{r}_2' \) considered fixed. In the most general case where all parameters in the seven-dimensional space coordinatized by \( \{ \mathbf{r}_1, \mathbf{r}_2; \omega \} \) are allowed to vary independently, coherence vortices will manifest as a series of five-dimensional sheets, at each point \( \{ \mathbf{r}_1, \mathbf{r}_2; \omega \} \) of which \( \mu \) vanishes.

Such nodal surfaces are generic rather than exotic. As has been emphasized in the book by Nye [6], the formation of coherent vortices is a ubiquitous and spontaneous means of pattern formation in coherent optical fields. Given that this ubiquity is ultimately based on topological arguments that rely solely on the continuity and single-valueness of complex functions of two or more real variables rather than on any specific details of the equations.
governing their evolution [7], similar considerations hold for coherence vortices.

As an example of the spontaneous formation of coherence vortices in optical scattering, consider the setup shown in Fig. 1. Here we see a z-directed polarized monochromatic electromagnetic plane wave that is incident upon three dielectric spheres. Under the space–frequency description of partial coherence [14], the incident field will be described at each frequency by an ensemble of monochromatic polarized plane waves, whose propagation vectors are randomly and uniformly distributed within a cone of specified cone angle that is coaxial with z—see Sec. 3 of Marasinghe et al. [13] for further details. Such a model for the incident illumination may be used to quantify the pointing instability of a high-power cw laser. Inspired by the fact that coherent vortices can be formed by the interference of three plane waves [16,17], and by the fact that both coherence vortices [12] and coherent vortices [18] can be formed in a Young-type three-pinhole interferometer, one is led to seek coherence vortices in the scattering of partially coherent plane waves by three coplanar dielectric spheres. This extends our previous work on the generation of coherence vortices by Mie scattering of partially coherent light, by a single dielectric sphere [13].

In Fig. 1, the origin of the coordinate system is taken as the center of the scattering sphere A. All three spheres have the same radius, \( R = 705.5 \, \text{nm} \), and refractive index, \( n = 3.6849 + 0.0087i \), with the wavelength of the incident light being \( \lambda_0 = R \). All spheres are placed in an equilateral triangle in the \( xy \) plane where \( z = 0 \), such that the distance between any two spheres is \( 13\lambda_0 \). At this separation, the singly scattered field intensity at a given sphere due to an adjacent sphere is less than 0.1% of the incident field. Hence multiple scattering effects may be neglected. To model partial coherence in the illumination, we considered an ensemble of 500 random monochromatic realizations of the incident planar electric field for all the calculations. The incident field's propagation vectors were randomly varied over a cone of half-apex angle, \( 1 \, \text{rad} \), in the ensemble of realizations. Under the space–frequency description of partial coherence, one can readily utilize the theory of coherent plane-wave Mie scattering to determine the spectral degree of coherence for scattering from a single dielectric sphere, as discussed in Marasinghe et al. [13]. Because multiple scattering may be neglected, we can apply the superposition principle to the results of Marasinghe et al. [13], to model the scenario shown in Fig. 1.

In our numerical simulations, \( \omega \) is considered fixed at \( 2\pi c/\lambda_0 \), with \( r_2 \) fixed at the origin of the coordinate system.

Figure 2(a) shows the magnitude of the spectral degree of coherence \( |\mu| \), with Fig. 2(b) showing the corresponding phase \( \arg(\mu) \). In both Figs. 2(a) and 2(b), the labels A, B, and C refer to the identically labeled points in Fig. 1. Both the magnitude and phase are displayed over the plane \( z = R \), i.e., over a planar exit surface tangential to the three scattering spheres. In Fig. 2(a) it can be seen that \( |\mu| \) takes values between zero and unity. Near the vicinity of the scattering spheres, the
field intensity has a higher value. In Fig. 2(b) we can see that the phase of $\mu$ exhibits a distorted lattice of coherence vortices and antivortices near the center of the observation plane (cf. [18]). For better visualization of the 3D structure of this coherence-vortex network, the evolution of $|\mu|$ and $\arg(\mu)$ along the direction of propagation is shown in Figs. 2(c) and 2(d), respectively. These figures show the coherence variation in a rectangular prism where $z \in [R, 10R]$. When $r_i$ and $\omega$ are considered fixed, the coherence vortices will be threaded by 1D nodal lines corresponding to the previously mentioned pairs of points at which the field is uncorrelated (at a given angular frequency). Figure 3 illustrates the nodal lines observed in a rectangular prism where $z \in [10R, 20R]$. A contour plot of the phase of the spectral degree of coherence has been projected onto the top and bottom surfaces of this rectangular prism, indicating that isophase surfaces converge on nodal lines. The nodal lines themselves are shown as thick solid lines. Some nodal lines demonstrate a hairpin structure—see the nodal lines shaded in pink (lower right) and brown (lower left). Nodal-line hairpins occur when a coherence vortex–antivortex pair nucleates or annihilates at a certain point (cf. [19]), by forming a single-bend continuous nodal line. The point at which nucleation or annihilation occurs is not well-defined, because it depends on how the contour foliates the space. We also observe coherence vortex–antivortex dipoles, with oscillations reminiscent of the dislocation helices reported in Berry et al. [20]—see the nodal-line pairs shaded in green (left), red (center), and blue (right).

We close with five remarks: (1) In addition to studying the screw-type topological defects in the spectral degree of coherence $\mu(r_1, r_2; \omega)$, it would be interesting to study higher-order topological defects in the matrix field $W^\mu(r_1, r_2; \omega)$. Such a study is suggested by the result that three-wave interference of multicomponent fields has previously been shown to yield higher-order (texture) topological defects [21]. (2) This Letter could be generalized to higher-order correlation functions, such as those employed in the theory of fourth-order partial coherence. (3) Given the results of the present Letter, together with the fact that coherent vortices may be generated by coherent light elastically scattered from a static random phase screen [22], one would expect a tangle of coherence vortices to be produced upon scattering of a partially coherent wave from such a screen. (4) Point (3) would also be interesting to explore in the context of the scattering of partially coherent electromagnetic radiation from biological tissue, e.g., the use of optical scattering to noninvasively sense blood glucose levels [23]. (5) It would be interesting to investigate the possibility of fractal behavior in the coherence-vortex networks in (3) and (4), along the lines developed by O’Holleran et al. [24] for coherent-vortex networks.

M. L. Marasinghe acknowledges financial support from both the Monash Research Graduate School and the Faculty of Engineering, Monash University.

References